AUGUST

NCTM
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

This month's "Calendar" is a relay calendar: In each row of the calendar, the answer to a problem will be used in the next consecutive question. The answer to problem n will be represented by A_n. This relay format is used in many competitions, including those of the American Regions Mathematics League.

1
On a reality show, teams of 6 players have to work together to accomplish a task. Each member of the team is tied to each of his or her teammates with a separate strand of rope. How many pieces of rope are required to tie together all 6 members of the team?

2
While sitting in his car at a train crossing, the driver decided to time how long it took for a 1-mile-long train to pass him entirely. If it took exactly $5 \cdot A_1$ seconds for the train to pass, at how many miles per hour was the train traveling?

3
In a local hotel there are A_2 steps from the first floor to the fourth floor. If we assume that the number of steps between consecutive floors is constant and that it takes 1.5 seconds to climb one step, how many seconds will it take to climb the stairs from the first floor to the tenth floor?

4
The sum of two integers is 28, and their product is 192. What are the two integers?

5
Let the two results called A_4 be the lengths of the diagonals of a rhombus. Determine the length of the radius of the circle inscribed in the rhombus.

6
Let $5 \cdot A_5$ be the measure of side ST in triangle $\triangle RST$ with $m\angle T = 30°$. Determine the number of integral values for side RS that will result in two distinct values for length RT.

7
Let $\log_k 2 = A_6/9$. Find $\log_8 k$.

8
Find the sum of the odd integers from 13 to 107, inclusive.

9
Determine the number of sides that a polygon must have if the sum of its interior angles, measured in degrees, is A_8.

10
The longest side of a triangle is A_9, and the shortest side is $A_9/2$. If the angle between the two known sides is 60°, find the length of the triangle's third side.

11
The numbers a and b are positive integers with $a < b$. Solve for a and b in the following equation:

$$a^3 + b^3 = A_{10}^{\ 2}$$

12
Driving west along U.S. Route 66, Kevin saw a sign that read: Amboy 128 miles, Needles 202 miles. Later, a second sign indicated that the distance to Needles was twice the distance to Amboy. Later still, a third sign indicated that the distance to Needles was three times the distance to Amboy. How many miles apart were the second and third signs?

13
Let A_{12} be the hypotenuse of a right triangle with integer legs. Find the positive difference between the two legs of the triangle.

14
Two streets mutually terminate, forming an angle of 52°. A tree is to be planted so that it is equidistant from the two streets and A_{13} ft. from the intersection. Find, to the nearest integer, the number of feet the tree will be from each street.

15
A game is played with a square board that is A_{14} ft. on a side. Pegs are placed at the four corners of the board. A ring whose radius is 1 ft. is tossed onto the board. If we assume that the center of the ring falls somewhere on the board, what is the probability that it encircles one of the pegs?

16
How many cubes, each 3 in. on a side, can fit in a rectangular prism whose dimensions are 2 ft. × 3 ft. × 4 ft.?

17
Let $p = A_{16}/100$ rounded to the nearest integer. Let $q = A_{16} - 100p$. The Yankees won p games out of the first q games played. What is the minimum number of additional games they would have to play to win 3/4 of the total number of games played?

18
In isosceles triangle DEF, $DE = EF = A_{17}$. Altitudes from E and F are drawn and intersect the opposite sides of the triangle at G and H, respectively. If $DH = 6$, find DF.

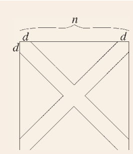
19
Determine the number of sides of a convex polygon that contains exactly $3.75 \cdot A_{18}$ diagonals.

20
What is the smallest integer that is divisible by all the integers 1 through 10, inclusive?

21
Let s = the sum of the digits in A_{20}, and let p = the product of the nonzero digits of A_{20}. Suppose that we have points J, K, L, and M on a circle with $JK = s$, $JM = p$, and the diameter $JL = s + p$. Perpendicular segments are drawn to JL from both K and M and intersect JL at E and F, respectively. Find EF.

22
Triangle $\triangle RED$ has point G on side RD. If $RE = 12$, $RG = 9$, $GD = 7$, and the perimeter of $\triangle EGD = 3A_{21} + 2$, determine the perimeter of $\triangle REG$.

23
Factor A_{22} into primes. Let n be the largest prime factor and d the smallest. A square is decorated with a symmetric X shape. If a side of the square is n and d is as needed, determine the area of the X.



24
It takes a person 6 days to cross a stretch of deserted land. One person can carry only enough food and water to survive for 4 days. What is the minimum number of people who must start out to ensure that one person successfully gets across and that the others safely return to the starting point?

25
Bonnie has coins that total more than $A_{24}/3$ dollars. Her friend asks her if she has change for a dollar bill and is surprised that she does not. What is the largest amount of money, in cents, that Bonnie can have and still not be able to give her friend change for a dollar?

26
The area of the base of a right rectangular pyramid is $100 \cdot A_{25} - 11$. If the lateral areas of two adjacent faces are 45 and $3\sqrt{97}$, compute the volume of the pyramid.

27
Suppose that for all $x \geq 0$,

$$f(g(x)) = \sqrt{x},\ g(f(x)) = x^2, \text{ and } g(12) = 25.$$

Find $g(A_{26})$.

28
When the waiter was asked to slice the 16-in. (in diameter) pizza into exactly 4 slices of equal size, he decided to do so using concentric circles rather than radial slices. Determine the radii of each of the three cuts he had to make.

29
Find the number of square units in the area of a triangle whose sides measure the three values of A_{28}.

30
Determine $17 \cdot A_{29}^{\ 2}$ and arrange the resulting digits in decreasing order. Then arrange the same digits in increasing order and subtract the smaller number from the larger. Repeat the process with the current result. Determine the number of times this process must be performed before you arrive at the number 6174 for the first time.

31
Given that

$$x^2 + 1/x^2 = A_{30},$$

evaluate the absolute value of

$$x^2 - 1/x^2.$$